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Article

Model Predictive Control Using State Estimation Based on Unscented Kalman Filter for Stabilization of Underwater Vehicle Dynamics

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Abstract: This research addresses the challenge of designing control systems to stabilize the nonlinear dynamics of underwater vehicles. Model Predictive Control (MPC) is a widely recognized technique that determines the current control input by solving an optimal control problem. However, MPC cannot be directly applied to systems where all state variables are not precisely known. Typically, state variables are inferred from sensor measurements, meaning that only a subset of them is available for control input design. This study aims to develop a control approach that stabilizes underwater vehicle dynamics by integrating a state estimation method into the MPC method. The novelty of this study is to develop a control framework that integrates MPC method with the state estimation method based on Unscented Kalman Filter (UKF) for stabilization of underwater vehicle dynamics.

Keywords: autonomous vehicle; underwater vehicle; nonlinear dynamics; optimal control; state estimation

1. Introduction

Autonomous underwater vehicles (AUVs) are anticipated to play a crucial role in a wide range of applications. The diverse utilization of these vehicles has driven researchers to advance control systems and technologies tailored for underwater operations [1–3]. Optimal control methods [4,5] have been proposed to reduce the control energy of AUVs. Multi-target tracking and sensing framework for AUVs have been investigated in research papers [6–8]. However, the nonlinearity and complexity of AUV's dynamics caused by fluid flow has not considered in [1–8]. This research addresses the challenge of designing control systems for stabilizing the nonlinear complex dynamics [9] of underwater vehicles.

Model Predictive Control (MPC), also referred to as receding horizon control [10–15], is a well-established approach in which control inputs are computed by solving an open-loop optimal control problem over a finite time horizon. This optimization is performed iteratively at each sampling instance. Therefore, MPC belongs to the class of optimal feedback control strategies that optimize control performance within a finite prediction window, where the initial and terminal times shift dynamically. MPC is widely regarded as one of the most effective control methodologies, as it facilitates optimal performance while incorporating constraints on state and control variables [16–19].

A method based on MPC has been proposed for stabilizing the nonlinear rotational dynamics of underwater vehicles [20]. However, the aforementioned control strategies cannot be directly applied to systems where all state variables are not fully observable. In practical scenarios, state variables are typically inferred from sensor outputs, meaning that only a subset of them is accessible for control input design. In other words, complete state information is often unavailable since measurements are limited by the capabilities of onboard sensors, restricting the number of directly observable variables. As a result, effective automatic control systems must integrate state estimation techniques. To address this issue, a state estimation framework for underwater vehicle dynamics with partially measurable state variables has been developed in [21] using the Unscented Kalman Filter (UKF). UKF is a well-recognized estimation algorithm that minimizes estimation errors while accounting for both process noise and sensor noise [22].

The primary objective of this study is to develop a control framework that integrates Model Predictive Control [20] with state estimation techniques [21] to enhance the stabilization of underwater vehicle dynamics. To achieve this, an observer system is introduced to estimate the state variables of underwater vehicle dynamics. Subsequently, the UKF-based state estimation technique is incorporated into the MPC framework. As a result, this study establishes a control methodology capable of handling underwater vehicle systems with limited state observability. To the best of our knowledge, no prior studies have proposed a model predictive control scheme that explicitly integrates state estimation in this manner. The efficacy of the proposed approach is validated through numerical simulations.

2. Notation and System Model

In this section, we introduce a system model of underwater vehicle dynamics. Figure 1 shows a schematic view of the system model of underwater vehicle. The notation utilized in the system formulation is summarized in Table 1.

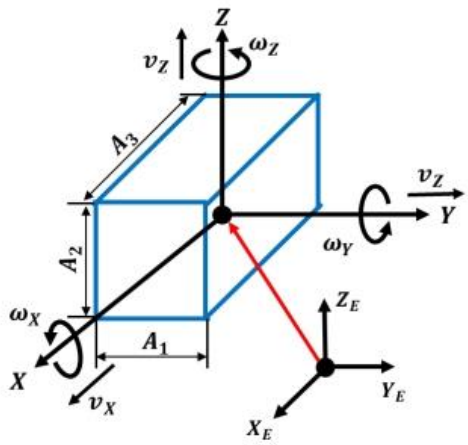

Figure 1. System model of underwater vehicle.

Table 1. Notations of System Parameters.

Notations	System Parameters
v_X, v_Y, v_Z	Translational Velocities
ω_X, ω_Y, ω_Z	angular velocities
m	vehicle mass
m_X, m_Y, m_Z	added mass
I_X, I_Y, I_X	principal moments of inertia
J_X, J_Y, J_X	added moments of inertia
u_1, u_2, u_3, u_4, u_5, u_6	control inputs

Let the state vector $x(t)$ be defined as

$$x(t) := [v_X(t), v_Y(t), v_Z(t), \omega_X(t), \omega_Y(t), \omega_Z(t)] \tag{1}$$

For the sake of simplicity in notation, the following parameters are introduced:

$$a_1 := m + m_X, \tag{2a}$$

$$a_2 := m + m_Y, \tag{2b}$$

$$a_3 := m + m_Z, \tag{2c}$$

$$b_1 := I_X + J_X, \tag{2d}$$

$$b_2 := I_Y + J_Y, \tag{2e}$$

$$b_3 := I_Z + J_Z. \tag{2f}$$

Using these notations, the system model shown in [20] can be described by the following state equation:

$$\dot{x}(t) = f(x(t), u(t)), \tag{3}$$

$$f(x(t), u(t)) := \begin{bmatrix} \frac{a_2}{a_1} x_3 x_6 - \frac{a_3}{a_1} x_3 x_5 + \frac{u_1}{a_1} \\ \frac{a_3}{a_2} x_4 x_4 - \frac{a_1}{a_3} x_1 x_6 + \frac{u_2}{a_2} \\ \frac{a_1}{a_3} x_5 x_5 - \frac{a_2}{a_3} x_2 x_4 + \frac{u_3}{a_3} \\ \frac{b_2 - b_3}{b_1} x_5 x_6 + \frac{u_4}{b_1} \\ \frac{b_3 - b_1}{b_2} x_6 x_4 + \frac{u_5}{b_2} \\ \frac{b_1 - b_2}{b_3} x_4 x_5 + \frac{u_6}{b_3} \end{bmatrix}.$$

The output function $y(t)$ is introduced as:

$$y(t) = Cx(t). \tag{4}$$

where C is a coefficient to account for restrictions on sensor allocation.

3. Model Predictive Control

In this study, the Model Predictive Control (MPC) approach is employed for the design of a control system for an underwater vehicle, considering the nonlinear characteristics of its rotational dynamics. Here, we formulate the optimal control problem associated with system (3).

The control input $u(t)$ is determined at each sampling instant by minimizing the following performance index:

$$J = \phi\big(x(t+T)\big) + \int_t^{t+T} L\big(x(\tau), u(\tau)\big) d\tau \tag{5}$$

Here, P, Q, and R represent weighting matrices, while T denotes the evaluation interval, commonly referred to as the prediction horizon. The functions ϕ and L correspond to the terminal cost and stage cost functions, respectively, and are defined as follows:

$$\phi = \big(x(t+T) - x_f\big)^{\mathrm{T}} P \big(x(t+T) - x_f\big)$$

$$L = \big(x(\tau) - x_f\big)^{\mathrm{T}} Q \big(x(\tau) - x_f\big) + u^{\mathrm{T}}(\tau) R u(\tau)$$

The minimization of (5) under the constraint imposed by the system equation (3) can be reformulated into an equivalent optimization problem by introducing a Lagrange multiplier λ, which accounts for the constraint associated with the system dynamics. Consequently, the following augmented performance index $\bar{J}$ is obtained:

$$\bar{J} = \phi\big(x(t+T)\big) + \int_{t}^{t+T} \big(L(x,u) + \lambda^{\mathrm{T}}(f - \dot{x})\big)d\tau \tag{6}$$

By applying the variational principle, the necessary conditions for minimizing $\bar{J}$ are derived as follows:

$$\dot{x} = f(x,u) \tag{7a}$$

$$\lambda(t+T) = \left(\frac{\partial\phi}{\partial x}\right)^{\mathrm{T}} \tag{7b}$$

$$\dot{\lambda} = -\left(\frac{\partial H}{\partial x}\right)^{\mathrm{T}} \tag{7c}$$

$$\frac{\partial H}{\partial u} = 0 \tag{7d}$$

These conditions, collectively referred to as the stationary conditions, are also known as the Karush-Kuhn-Tucker conditions or Euler-Lagrange equations. It is well established that these conditions must be satisfied to achieve the minimization of the performance index (6). In general, solving these conditions analytically is not feasible. Recently, various numerical techniques have been introduced to address this problem. The Continuation/Generalized Minimum Residual (C/GMRES) method, proposed in [23], is one such efficient numerical algorithm. In this study, we utilize the C/GMRES method to solve the stationary conditions in (7). A comprehensive explanation of the C/GMRES method can be found in [23].

4. State Estimation based on Unscented Kalman Filter

Let τ represent the discrete-time index corresponding to the sampling interval Δt. The continuous-time system model given in (3) can be transformed into the following discrete-time representation:

$$x(\tau+1) = \hat{f}\big(x(\tau), u(\tau)\big) \tag{8a}$$

$$y(\tau) = C\big(x(\tau)\big). \tag{8b}$$

In this section, we introduce a state estimation approach utilizing the UKF for the discrete-time system model (8). To begin, we define the observer system as follows:

$$\hat{x}(\tau+1) = \hat{f}\big(\hat{x}(\tau), u(\tau)\big) + z(\tau), \tag{9a}$$

$$\hat{y}(\tau) = C\hat{x}(\tau) + w(\tau)\#(9b) \tag{9b}$$

Here, $\hat{x}$ and $\hat{y}$ denote the estimated state and output, respectively, corresponding to x and y. The terms z and w represent the process noise and observation noise, which originate from external disturbances. In the framework of minimum mean-squared error estimation, the optimal state estimate is given by the conditional mean. It is assumed that both $z(\tau)$ and $w(\tau)$ have zero mean for all time steps τ.

It has been established that the state estimate at time $\tau+1$ is obtained by refining the prediction through a linear update rule:

$$\mathbf{K}(\tau + 1) = \mathbf{P}(\tau + 1|\tau)\mathbf{R}^{-1}(\tau + 1|\tau), \tag{10a}$$

$$\hat{x}(\tau+1|\tau+1) = \hat{x}(\tau+1|\tau) + \mathbf{K}(\tau+1)\big(\bar{y}(\tau+1) - \hat{y}(\tau+1|\tau)\big), \tag{10b}$$

$$\mathbf{Q}^{\hat{x}}(\tau+1|\tau+1) = \mathbf{Q}^{\hat{x}}(\tau+1|\tau) - \mathbf{K}(\tau+1)\mathbf{R}(\tau+1|\tau)\mathbf{K}^{\mathrm{T}}(\tau+1). \tag{10c}$$

The state estimation framework presented in this section is integrated into the Model Predictive Control method described previously. A brief overview of the numerical solution method, which combines the C/GMRES approach with the state estimator based on UKF is illustrated in Figure 2.

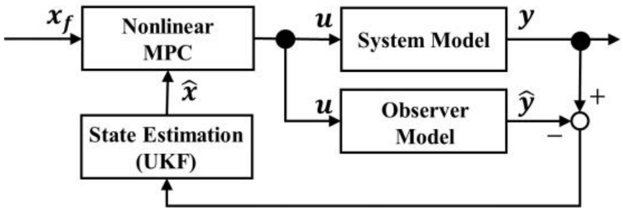

Figure 2. Overview of the proposed method.

5. Numerical Simulations

This section presents the results of numerical simulations aimed at validating the effectiveness of the proposed method. The parameters used in the simulations are listed in Table 2. Notably, in these simulations, x_1 is not measured through the output sensors.

Two numerical simulations are performed with different initial state estimates to evaluate the method's performance under varying conditions. The error e_i is defined as $e_i = x_f - x_i$. The first simulation uses the following initial estimate:

$$\hat{x}(0) = \left[2, 1, 1, \frac{\pi}{36}, \frac{\pi}{36}, \frac{\pi}{36}\right]^{\mathrm{T}}$$

In the second simulation, a different initial estimate is used:

$$\hat{x}(0) = \left[3, -2, 3, \frac{\pi}{12}, \frac{\pi}{12}, -\frac{\pi}{36}\right]^{\mathrm{T}}$$

Figures 3 and 6 show the time responses of $x(t)$ and $\hat{x}(t)$. It can be seen from these figures that the estimated state converges to the real state, and the real state converges to the target state. Figures 4 and 7 show the time responses of the errors between the target states and the actual states. Additionally, Figures 5 and 8 show the time responses of the control inputs. Consequently, the effectiveness of the proposed method was verified by numerical simulations.

Simulation is performed on a laptop computer (CPU: Intel(R) Core(TM) i5-1135G7 2.4 [GHz], Memory: 16.0 [GB], OS: Windows 11 Pro, Software: Matlab). The average computational time including both MPC and UKF computations per update (one control cycle) is 1.4 [ms]. The increase in computational load due to the number of steps in the evaluation interval or the number of the state variables is not exponential but less linear growth.

Table 2. Parameters used in numerical simulation.

Parameters	Parameters
$(0) = \left[1, 1, 1, \frac{\pi}{36}, \frac{\pi}{36}, \frac{\pi}{36}\right]^{\mathrm{T}}$	$\mathbf{Q}^z = \mathrm{diag}\,[0, 0, 0, 0, 0, 0]$
$x_f = [0, 0, 0, 0, 0, 0]^{\mathrm{T}}$	$\mathbf{Q}^w = \mathrm{diag}\,[0.1, 0.1, 0.1, 0.1, 0.1, 0.1]$
$I_X = 0.438$	$m_X = J_X = 34.9$
$I_Y = 0.833$	$m_Y = I_Y = 101$
$I_Z = 0.758$	$m_Z = J_Z = 82.5$

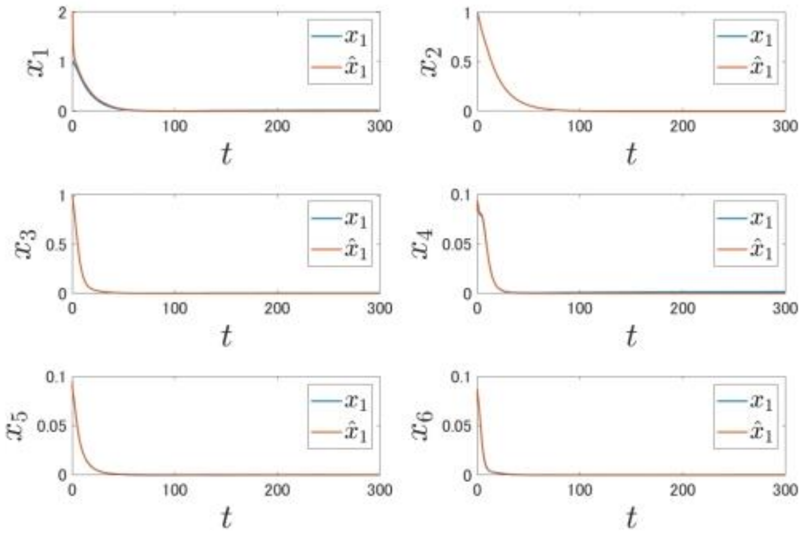

Figure 3. Time responses of $x(t)$ and $\hat{x}(t)$.

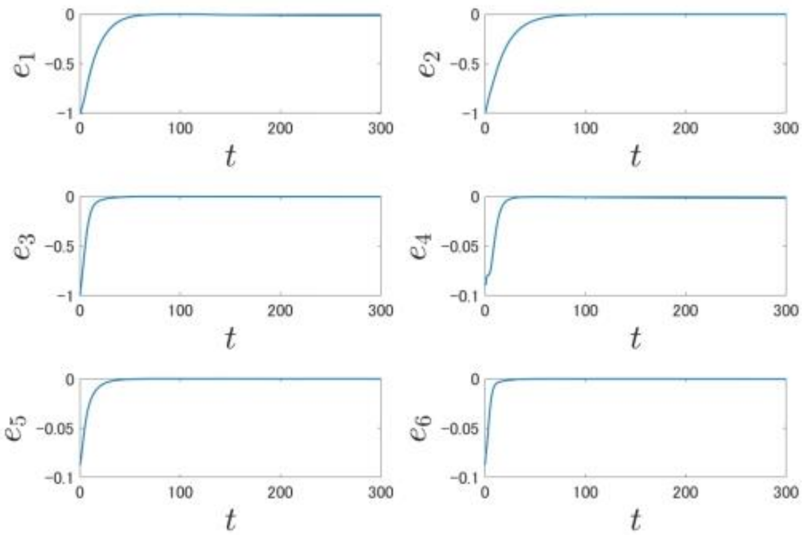

Figure 4. Time responses of $e(t)$.

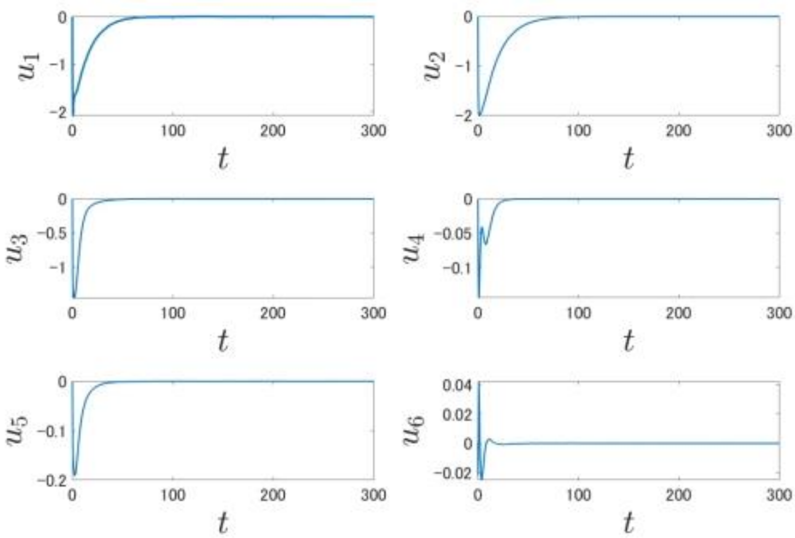

Figure 5. Time responses of $u(t)$.

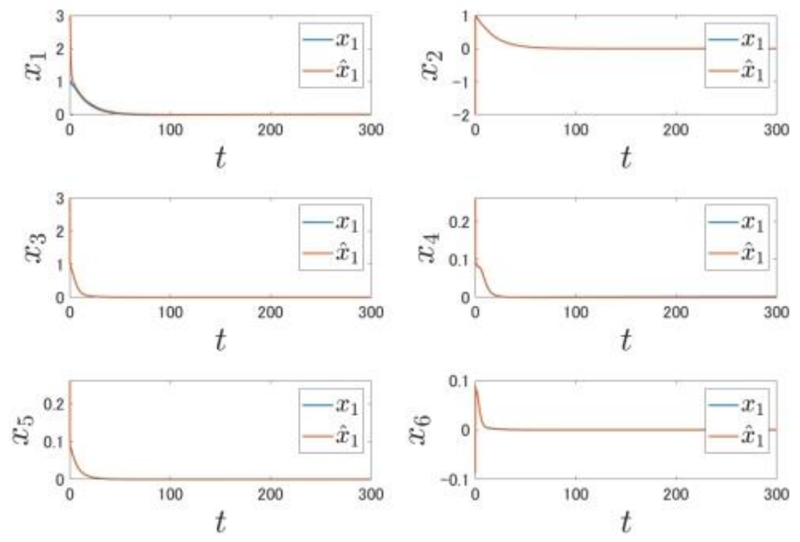

Figure 6. Time responses of $x(t)$ and $\hat{x}(t)$.

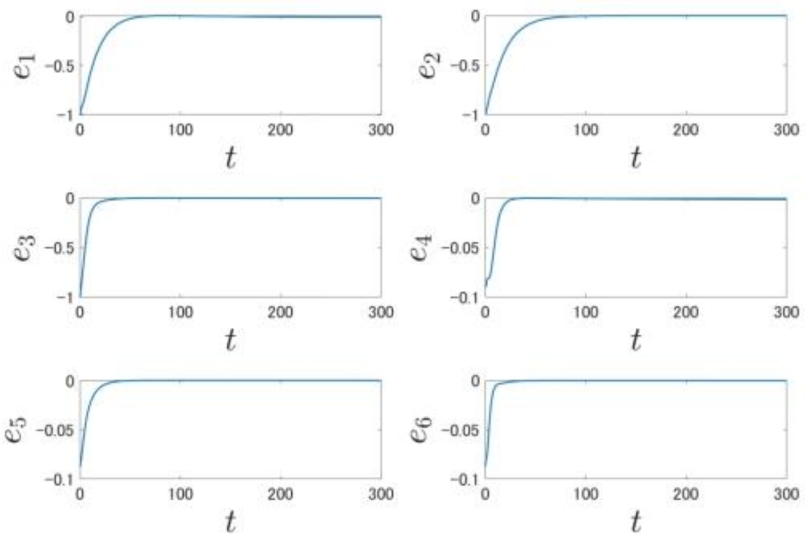

Figure 7. Time responses of $e(t)$.

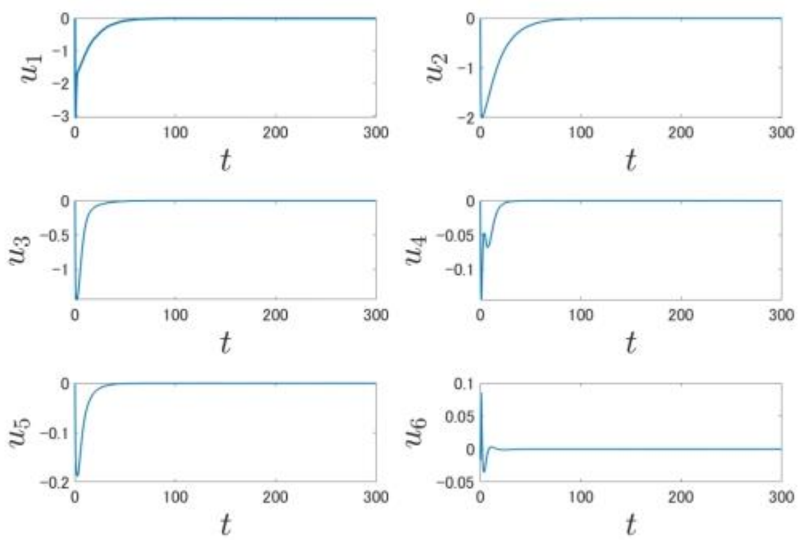

Figure 8. Time responses of $u(t)$.

6. Conclusion

In this study, we propose a control method for stabilizing underwater vehicle dynamics, considering the scenario where some state variables of the system are unobservable, and the sensor output is contaminated with observation noise. The approach applies MPC with a fast numerical algorithm, C/GMRES, and a state estimation method based on the UKF to design the control system. The MPC method previously proposed for controlling underwater vehicles cannot be applied when not all state variables of the system are exactly known. In practice, state variables are typically measured through output sensors, meaning that only a limited subset of them can be used to design the control inputs. To address this, we propose a control method that combines MPC with a state estimation technique for stabilizing underwater vehicle dynamics. Finally, we present the results of numerical simulations that validate the effectiveness of the proposed method.

In this study, it is assumed that the process noise and observation noise are zero-mean and Gaussian. In real-world scenarios, noise does not always follow these assumptions. In the case of non-Gaussian or correlated noise conditions, the estimation method modified using particle filter might be more effective. This investigation is considered to be a possible future work.

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